Carleton University
Department of Systems and Computer Engineering

SYSC 5401 **Adaptive and Learning Systems** **Winter 2025**
Assignment #2

Due Date: **Wednesday February 12, 2025**

Question 1: The Instrumental Variable Approach

Simulate the following system:

$$y(k) - 1.5y(k-1) + 0.7y(k-2) = u(k-1)$$

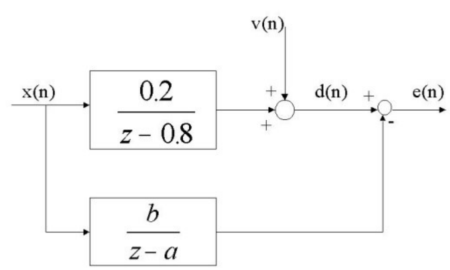
and a measurement signal

$$y_m(k) = y(k) + e(k)$$

Where *e(k)* is a white noise. Create data sets where *e*(*k*) has a variance of 4.0 and 9.0. The input, *u(k)* is a random binary signal with *u(k)* = ±1. Create the data sets with N = 400 points. One can only use the measurement signal $y_m(k)$ for estimating the parameters. Use the instrumental variable approach to identify the parameters. Show at least three iterations of the instrument selection and parameter estimation procedure. What is the cost function for each case? Plot the output and the predicted output for each iteration on the same graph.

Question 2) Steepest Descent, LMS and RLS

You are given the following systems identification problem:



The input x(n) is a white noise process with mean E[x(n)] = 0 and variance $E[x^2(n)] = 1.0$.

a) Assume $v(n) = 0$. Simulate the system using the steepest descent algorithm, the LMS algorithm and the standard recursive least squares algorithm with forgetting factor $\lambda = 1.0$. Plot the estimated parameters as a function of time and on the a-b plane. On the same plot show the contours of the MSE surface. Initialize a=b=0.

b) Comment on the difference in performance, complexity and implementation issues.

Question 3: Recursive Least Squares

Simulate the following system:

$$y(k) - 1.5y(k-1) + 0.7y(k-2) = u(k-1) + e(k)$$

Where *e(k)* is a white noise with variance 4.0 and *u(k)* is a random binary signal with *u(k)* $= \pm 1$. Simulate the data set with N = 400 points.

a) Use the obtained data sets of (y, u) from part a) to identify the parameters using the recursive least squares algorithm. Plot the parameter estimates and the trace of the covariance matrix *P*. Let the forgetting factor $\lambda = 1.0$ and 0.95. Is there a difference?

b) Do the same simulation as in part a) but set *u(k)* = +1.0 from N=100 to N=300 and then change *u(k)* back into a random binary signal, *u(k)* $= \pm 1$ from N=301 until N=400. Set the forgetting factor to $\lambda = 0.95$. Plot the parameter estimates and the trace of the covariance matrix *P*. Any comments?

c) Do the same simulation as you did in part b) except this time change the parameter values at time step N=250 from $a_1 = -1.5$ to $a_1 = -1.7$ and change $a_2 = 0.7$ to $a_2 = 0.9$ and change $b = 1.0$ to $b = 5.0$. Comment on what you observe. Plot the parameter estimates and the trace of the covariance matrix.